Math 10 Lecture Videos

Section 4.2:

Solving Systems of Linear Equations by the Substitution Method

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OBJECTIVES:

1. Solve linear systems by the substitution method.
2. Use the substitution to identify systems with no solution or infinitely many solutions.
3. Solve problems using the substitution method.

Objective 1: Solve linear systems by the substitution method.

1. Solve either of the equations for one variable in terms of the other. (If one of the equations is already in this form, you can skip this step.)
2. Substitute the expression found in step 1 into the other equation. This will result in an equation in one variable.
3. Solve the equation containing one variable.
4. Back-substitute the value found in step 3 into the equation from step 1. Simplify and find the value of the remaining variable.
5. Check the proposed solution in both of the system’s given equations.

Objective 1: Solve linear systems by the substitution method.

Example: Solve the system using the substitution method:

$$y = 2x - 8$$
$$2x + 3y = 16$$

1. Solve either of the equations for one variable in terms of the other.

Note that the first equation is already solved for *y*, so we will use that.

2. Substitute the expression from step 1 into the other equation.

$$2x + 3(2x - 8) = 16$$

Substitute $2x - 8$ for y

Objective 1: Solve linear systems by the substitution method.

Example: Solve the system using the substitution method:

$$y = 2x - 8$$
$$2x + 3y = 16$$

3. Solve the equation containing one variable.

$$2x + 3(2x - 8) = 16$$
$$2x + 6x - 24 = 16$$
$$8x - 24 + 24 = 16 + 24$$
$$8x = 40$$
$$x = 5$$

4. Back-substitute the value found in step 3 into the equation from step 1.

$$y = 2x - 8$$
$$y = 2(5) - 8$$
$$y = 10 - 8$$
$$y = 2$$

The proposed solution is $x = 5$ and $y = 2$ or the ordered pair (5,2)

Objective 1: Solve linear systems by the substitution method.

Example: Solve the system using the substitution method:

$$y = 2x - 8$$
$$2x + 3y = 16$$

5. Check the proposed solution in both of the system's given equations.

Equation 1

$Y = 2x - 8$
$2 = 2(5) - 8$
$2 = 10 - 8$
$2 = 2$

Equation 2

$2x + 3y = 16$
$2(5) + 3(2) = 16$
$10 + 6 = 16$
$16 = 16$

Ordered pair (5,2) satisfies both equations. It is the system's solution.

Objective 2: Use the substitution to identify systems with no solution or infinitely many solutions.

Using the Substitution Method on an **Inconsistent System**

A linear system with no solution is called an **inconsistent** system.

When solving such a system by substitution, both variables are eliminated.

A *false* statement (such as $0 = 5$) will be the result.

Objective 2: Use the substitution to identify systems with no solution or infinitely many solutions.

Using the Substitution Method on an **Inconsistent System**

Example 1: Solve using the substitution method:

$3x + y = -5$

$y = -3x + 3$

Since the second equation is solved for *y*, substitute -3x + 3 for *y* in the first equation.

$3x + y = -5$

$3x + (-3x + 3) = -5$

$3x - 3x + 3 = -5$

$3 = -5$

False!

The false statement indicates that the system is inconsistent and has no solution. The solution set is { }.

Objective 2: Use the substitution to identify systems with no solution or infinitely many solutions.

Using the Substitution Method on a **System with Infinitely Many Solutions.**

Equations in a linear system with infinitely many solutions are called **dependent**

When solving such a system by substitution, both variables are eliminated.

A *true* statement (such as 6 = 6) will be the result.

Objective 2: Use the substitution to identify systems with no solution or infinitely many solutions.

Using the Substitution Method on a **System with Infinitely Many Solutions.**

Example 1: Solve using the substitution method:

$y = 3x - 4$

$9x - 3y = 12$

Since the first equation is solved for *y*, substitute 3x – 4 for *y* in the second equation.

$9x - 3y = 12$

$9x - 3(3x - 4) = 12$

$9x - 9x + 12 = 12$

$12 = 12$ **True!**

The true statement indicates that the system contains dependent equations and has infinitely many solutions.

Solution Set: $\{(x,y) \mid y = 3x - 4\}$ or $\{(x,y) \mid 9x - 3y = 12\}$.

Objective 3: Solve problems using the substitution method

Example: The following models describe demand and supply for two-bedroom rental apartments, where p is the monthly rental price and x is the number of apartments.

Demand Model: $p = -30x + 1800$

Supply Model: $p = 30x$

Solve the system and find the equilibrium quantity and the equilibrium price.

Note: Substitute 30x for p in the first equation.

$$p = -30x + 1800$$
$$30x = -30x + 1800$$
$$30x + 30x = -30x + 1800 + 30x$$
$$60x = 1800$$
$$x = 30$$

Objective 3: Solve problems using the substitution method

Example: The following models describe demand and supply for two-bedroom rental apartments, where *p* is the monthly rental price and *x* is the number of apartments.

Demand Model: $p = -30x + 1800$

Supply Model: $p = 30x$

Back-substitute to find p.

$p = 30x$

$P = 30(30)$

$P = 900$

Equilibrium Quantity

The solution set: {(30, 900)}

Equilibrium Price

When rents are $900 per month, consumers will demand 30 apartments and suppliers will offer 30 apartments for rent.

OBJECTIVES:

1. Solve linear systems by the substitution method. ✓
2. Use the substitution to identify systems with no solution or infinitely many solutions. ✓
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